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A Note on the Impossibility of Local Pareto Satiation in an Economy with Externalities

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Abstract

In this note, we argue that structure of production sets for markets with externalities precludes the possibility of interior points being optimal. Even in an economy in which the externalities are not classical is structure an ordinary monotonicity assumption is sufficient to guarantee that all optimal production plans are an the boundary of the production sets. Thus, Starrett's assumption of no Local Pareto Satiation is not necessary. This focuses attention on the real reason that we can not count of decentralizing production of an optional plan through prices in such an economies: the fundamental nonconvexities identified by Starrett (1974).

Starrett's 1974 paper "Fundamental Nonconvexities in the Theory of Externalities" is one of the important contributions to the theoretical understanding of this classical problem. His approach is to define the extended Arrow commodity space associated with an economy with externalities, and then to examine the properties of the production set. Recall this extended space is obtained by introducing a set of "artificial commodities" which may be thought of as each agents' observation of the each other agents' consumption or production level of each good. In his second section he demonstrates very clearly that there is little hope of ever supporting optimal production plans in the extended space due the a "fundamental nonconvexity" that the presence of externalities introduces. The point that externalities and convex production sets are essentially incompatible is extremely important and (I think) not widely enough appreciated in Public Finance circles.

I do not take issue with the Starrett's conclusions about fundamental nonconvexities in this comment, but rather the point he raises in the first section of his paper. Starrett notes that even in an appropriately defined Arrow commodity space, the optimal production plan must be on the boundary of the production set if there is to be possibility of supporting it by a price system. He worries there might be a "keeping up with the Jones" effect that results in interior points of the production set being optimal. He calls this effect "local Pareto satiation" and argues that "Non-Local Pareto Satiation" (NLPS) is a necessary condition for the existence of prices supporting an optimal production plan. I argue here that NLPS is always satisfied as a consequence of the structure of the Arrow commodity space. Thus, while Starrett's conclusions are perfectly true, the assumption of NLPS in the hypothesis of his theorems is redundant.

The problem that Starrett has in mind can be easily illustrated with a simple example. Consider an economy with only two agents: Jones and Smith, and two goods: acres of lawn for Jones and acres of lawn for Smith. Denote these two goods as X^J, and X^S. Let the feasible allocations for these two goods be described by

$X^J \leq 4 - X^S$. Starrett imagines a situation in which each of these two goods has an associated negative externality. This is because Smith envies Jones' lawn and inversely. This external effect might be so strong that even though two acres of lawn for each agent is a feasible allocation, they would both be better off if they each had only one acre of lawn. This is despite the fact that for each agent, having a bigger lawn is always better for any fixed size of his neighbor's lawn. An arms race might be a real world example of this effect. In Starrett's language, one acre of lawn each is a point of local Pareto satiation. He concludes that since no prices could possibly support this interior Pareto efficient allocation, NLPS must be assumed to guarantee that all optional plans are one the boundary of the feasible set.

There are two reasons NLPS is an unnecessary assumption. First, recall that we are looking for supporting prices in the extended Arrow commodity space, not in the original goods space. If this extended space is defined in the traditional way, two goods are added to the example. These are Smith's observation of Jones' lawn and Jones' observation of Smith's lawn. Let the four commodities be denoted by X_j^i where $i, j \in \{S, J\}$, and this is interpreted as j's observation of i's lawn. For the standard case of externalities, observations are jointly produced. The extended production set can be described by three expressions: $X_J^J \leq 4 - X_S^S, X_S^S = X_J^S, X_J^J = X_S^J$. Here is the essential point: these equations describe a 2-dimensional manifold in a 4-dimensional space. In general, If there are N goods, F firms, and C consumers, the feasible set will always be an N-dimensional manifold in an $N(F+C)^2$-dimensional Arrow commodity space. Since such a manifold can never have an interior in a larger dimensional space, it follows that regardless of which production plan is optimal, it will necessarily be on the boundary of the extended production set. This means that it will be supportable by prices under the right convexity conditions. Of course, these are prices for all the commodities. It will still not generally be possible to support the optimum in the original production space since the projection of the optimal plan from the

extended space may very well be in the interior of the original space. But this is of no consequence.

Suppose instead that we want to consider something not quite as rigid as the classical externality model. In particular, suppose that for some reason outputs and observations of outputs are not strictly jointly produced. For example, what if it were possible to costlessly build a fence between the lawns of Smith and Jones that would partially or fully obscure observation of the other's lawn. In this case the expressions describing the production possibilities become $X_J^J \leq 4 - X_S^S, X_S^S \geq X_J^S, X_J^J \geq X_S^J$. The production set is no longer a submanifold of the Arrow commodity space and an such has an interior. Even here there is a reason that NLPS is not required. All that we need to prevent the possibility of an interior optimum is a standard monotonicity assumption. This will ordinarily be satisfied in such a commodity space, and in any event, is no stronger than what is needed for economies without externalities. This is much weaker that the sufficient condition that Starrett offers to guarantee a boundary optimum, namely that there be one good (like money) that in which all agents have monotonic preferences, and which may be consumed with out generating any externalities. In our example, all agents have weekly monotone preferences over all four goods. The optimal allocation in this case involves somehow deciding on an exact division of the four acres of lawn between the two agents, and then building the fence. In this case, of course, we are essentially freely disposing of the externality. This obscures the externality aspect of the problem to some degree. Still the point remains that monotonicity on the extended space is sufficient insure that all Pareto optimal production plans are on the boundary of the extended production sets.

References

Starrett, David (1972): "Fundamental Nonconvexities in the Theory of Externalities," *Journal of Economic Theory*, pp. 180-199.